Dynamical Interactions and the Black Hole Merger Rate of the Universe

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Binary black holes can form efficiently in dense young stellar clusters, such as the progenitors of globular clusters, via a combination of gravitational segregation and cluster evaporation. We use simple analytic arguments supported by detailed N-body simulations to determine how frequently black holes born in a single stellar cluster should form binaries, be ejected from the cluster, and merge through the emission of gravitational radiation. We then convolve this "transfer function" relating cluster formation to black hole mergers with (i) the distribution of observed cluster masses and (ii) the star formation history of the universe, assuming that a significant fraction $g_{\rm cl}$ of star formation occurs in clusters and that a significant fraction $g_{\rm evap}$ of clusters undergo this segregation and evaporation process. We predict future ground–based gravitational wave (GW) detectors could observe $\sim 10^4 g_{\rm cl} g_{\rm evap}$ double black hole mergers per year. Under the most optimistic assumptions ($g_{\rm cl} = g_{\rm evap} = 1$), the presently operating LIGO interferometer could detect a merger during its first full year of science data; upper limits therefore weakly constrain star and cluster formation in the early universe.

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Given our understanding of how isolated binary stars evolve, noninteracting stellar systems should produce relatively few double black hole (BH-BH) binaries tight enough to merge through the emission of GW within the age of the universe [1, 2, 3]. Portegies Zwart and McMillan [4] demonstrated that *interactions* between black holes (BHs) in dense cluster environments could produce merging BH-BH binaries much more efficiently than through the evolution of isolated binaries. As a result, the local binary black hole merger rate – the net rate both from isolated evolution of noninteracting stars and from dense clusters – can depend sensitively on the formation and evolution of young clusters through the entire history of the universe.

An increasing number of galactic [5, 6] and extragalactic [7, 8] observations suggest at least 20% of stars form in dense, interacting stellar clusters. Over time, each cluster dissipates, both because hot young stars and supernovae (SN) heat and eject a significant fraction of the residual gas that gravitationally binds the cluster ("infant mortality"), and because the host galaxy's tidal field strips off stars as the cluster orbits it [9, 10]. Thus any set of coeval clusters decrease in number and size, spewing stars into their hosts [11, 12], with only a few of the most initially dense and orbitally-fortunate clusters surviving to the present.

Unfortunately, electromagnetic observations of large young clusters in other galaxies cannot resolve their internal structure. These observations therefore only weakly constrain the fraction of young clusters that survive their first few Myr, during which the most massive young stars evolve, supernovae, and give birth to black holes. Similarly, they cannot determine the efficacy of *runaway stellar collisions*, in which massive stars in particularly dense clusters gravitationally segregate and collide to form very massive stellar progenitors to large [$\gtrsim 100\,M_\odot$] single or binary black holes [13, 14, 15, 16, 17]. However, clusters give birth to many binary black holes, whose GW merger signals provide unambiguous information about the processes which produced them. For example, Fregeau et al. [13] demonstrated that runaway stellar collisions produce very massive [$\sim 100\,M_\odot$] double BH binaries often enough to be easily and unambiguously seen with advanced ground-based interferometers (i.e., advanced LIGO/VIRGO). The detection rate of such massive BH mergers would therefore constrain the fraction of young clusters which undergo collisional runaway. Similarly, in this Letter we argue that a high GW detection rate of $\sim 15-20\,M_\odot$ black holes will unambiguously measure how often young clusters dynamically evaporate their most massive compact components. Specifically, we use the results of a recent set of numerical simulations of BHs in clusters [18] to present a simple, analytic approximation relating the BH-BH merger rate to properties of young clusters.

Within each cluster, stars form according to an approximately power-law mass distribution [19], with only a small number $N_{bh} \approx 3\times 10^{-3}(M_{cl}/M_\odot)$ of stars more massive than $20M_\odot$, roughly the mass needed to guarantee a black hole forms in a SN . These most massive stars should rapidly evolve, undergo SN , and form black holes on a timescale much shorter than the relaxation time of the cluster. The most massive clusters with $M_{cl} > M_{\rm crit} \equiv 3\times 10^4$ should contain more than 100 such stars, each of which produces a roughly $m_{bh} \approx 10-25M_\odot$ hole [20]. Being the most massive objects in the cluster, systems (single or binary) containing BHs will rapidly mass segregate to the cluster core [15, 21]. In the process exchange interactions will quickly break up any remaining star-BH binaries [22]. Since the numerous black holes significantly outweigh the average constituents of the cluster ($\langle m_* \rangle \approx 0.5 M_\odot$), the black holes can decouple from the stars in a process known as the "Spitzer instability" [23, 24]. The more massive BHs interact and evolve on a more rapid timescale $\approx t_{cl} \langle m_* \rangle / \langle m_{bh} \rangle$, quickly evaporating and ejecting single and binary BHs. Thermal equilibrium with the surrounding stellar cluser is only restored when so few BHs remain that the subcluster's internal timescale once again becomes commensurate with the cluster's interaction timescale. This process of segregation, decoupling, and evaporation has been extensively studied theoretically [25, 26] and numerically, both in full N-body simulations ($N \approx 10^3 - 10^5$) [4, 27, 28, 29] and in using approximations in larger N systems [15, 18].

Previous studies have suggested that interactions in this dense cluster can produce many BH-BH mergers, whether through evaporated binaries [4] or through runaway BH-BH mergers in the dense cluster itself [30, 31, 32]. However, these studies have faced significant objections to their details, because each has omitted some feature which could significantly inhibit the channel suggested. For example, the GW recoil produced during a BH-BH merger likely ejects the products of such a merger from any protocluster and prevents BH-BH merger runaway; the characteristic GW merger time of evaporated BH binaries depends sensitively ($\propto \sigma^8$) on the inital cluster velocity dispersion; and even the initial cluster's dynamics differs significantly depending on whether a discrete (two-component) or realistic (continuous) mass distribution is used.

For this reason, O'Leary et al. [18] performed a broad range of detailed numerical simulations of the dynamics of the segregated core and its coupling to the surrounding cluster, incorporating several critical physical features of dense cluster dynamics (a realistic black hole mass distribution; a range of BH-BH merger kicks that includes recoil speeds larger than the most extreme cluster escape velocities, as suggested by the most recent numerical simulations of unequal-mass binaries [33, 34]; a range of cluster velocity dipersions σ from 5 to $20\,\mathrm{km\,s^{-1}}$, encompassing the range seen in present-day globular clusters; and both three-body and four-body interactions). These simulations form the basis of our estimates of the numbers and properties of the evaporated BH-BH binary population ($N_{\mathrm{bin}} \approx 0.07 N_{\mathrm{bh}}$) that is ejected during the decoupled phase, consistent with theoretical expectations [25, 26]. These "hot" ejected binaries have a thermal eccentricity (e) distribution ($2ede$) and a lognormal binding energy ($E_b = G m_1 m_2 / 2a$) distribution, with half the binaries having binding energies between $10^{3.5}$ and $10^{4.5}$ times the thermal energy of the core, $kT = \langle m_* \rangle \sigma^2/3$ (see Fig. 6 of O'Leary et al.). Once ejected, the BH-BH

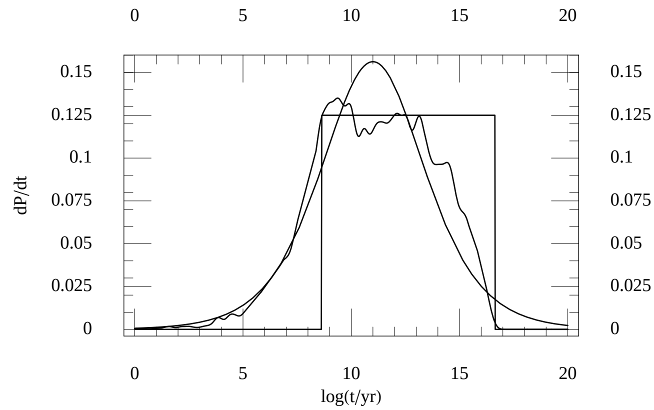

FIG. 1: The distribution of GW merger timescales for circular (solid step function) and eccentric (solid smooth function, based on 10^4 samples) $14M_\odot + 14M_\odot$ binaries, assuming the binding energy is distributed logarithmically from $10^3 k_b T$ to $10^5 k_b T$ for $k_b T$ the characteristic cluster energy. This figure assumes $k_b T = 0.5 M_\odot (10\,\mathrm{km\,s^{-1}})^2/3$.

binaries' orbits decay through GW emission, according to Peters' Eqs. (5.6-7) [35], until eventually each binary merges. The initially most strongly bound and eccentric orbits merge first, followed by the initially widest and most circular [Fig. 1], with a GW delay time $t_{gw} \propto a^4$. Furthermore, because of the broad range of ejected binary energies, while the differential distribution's *characteristic* merger time depends sensitively on the assumed cluster velocity dispersion ($t_{gw} \propto a^4 \propto \sigma^{-8}$) and to a lesser extent on the precise details of the energy spectrum of ejected binaries, the delay time distribution between 0.1 and 13 Gyr is comparatively robust, varying by $\sim$ 50%. Specifically, when a population of $m_{bh} = 14 M_\odot$ black holes evaporates from inside a cluster with a $\sigma_{\mathrm{cl}} = 10\,\mathrm{km\,s^{-1}}$ velocity dispersion, the binary black holes will have a probability $P(<t)$ of merging by time t, with

$$\frac{dP}{dt}(t) \approx \frac{2.5}{16t\log(10)} \mathrm{sech}\left[\frac{2.5(\log(t)-11)}{8}\right], \quad (1)$$

where we simplify the distribution of binding energies by assuming that it is flat in the log; a similar expression applies to general velocity dispersions σ_{cl}, when the logarithm (only) is rescaled according to $\log t \rightarrow \log t (\sigma_{\mathrm{cl}}/10\,\mathrm{km\,s^{-1}})^8$. Except for very early and very late times, this distribution is well approximated by $dP/dt \approx 0.054/t$, as expected from the relation between GW delay time ($t_{gw} \propto a^4$) and the logarithmic distribution of semimajor axes implied by cluster evaporation. This simple $1/t$ decay in merger rate per cluster appears in the simulations of O'Leary et al. (see their Figure 5).

Combining the number of black holes we expect per unit mass ($N_{bh} \approx 3 \times 10^{-3} (M_{cl}/M_\odot)$), the fraction of those black holes which should be ejected as binaries ($N_{\mathrm{bin}} \approx 0.07 N_{\mathrm{bh}}$, based on a variety of simulations [18]),

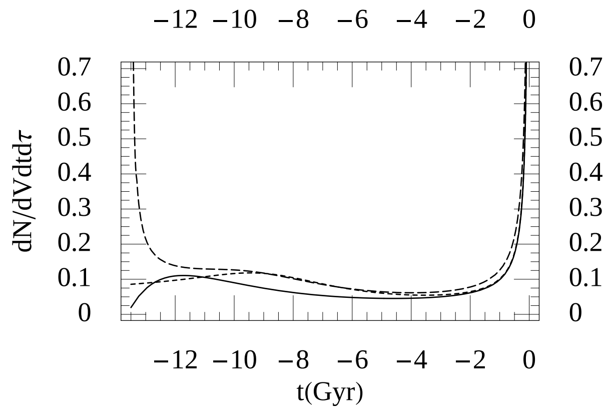

FIG. 2: The distribution of binary birth times for those binary black holes merging at present, given three different assumptions for the star formation history in the universe: Nagamine et al. [37] (solid), Porciani and Madau's Eq. (5) (dotted), and (6) (dashed) [38]. In all three cases, clusters formed more than 5 Gyr ago produce more BH-BH mergers than more recent cluster formation.

and the rate of mergers per unit time given by Eq. (1), we expect the number of cluster mergers per unit cluster mass after a time t to be $\mathcal{R}_{cl}(t) = 2.1 \times 10^{-4} M_\odot^{-1}\, dP/dt$. Assuming that clusters are formed from a fraction $g_{\rm cl}$ of available star formation and that only a fraction $g_{\rm evap}$ of all cluster-forming mass possesses the birth conditions necessary for this process to occur, then the BH-BH merger rate $R_{\rm evap}(t)$ per unit comoving volume is

$$R_{\rm evap}(t) = \int^t d\tau g_{\rm cl} g_{\rm evap} \mathcal{R}_{cl}(t-\tau)\frac{d\rho}{dt}(\tau)\,. \qquad (2)$$

where $d\rho/dt$ is the observed star formation rate per unit volume in the universe [36, 37]. While observations have yet to converge to a unique star formation history, most models for $d\rho/dt$ imply a present-day merger rate within 30% of $\approx g_{\rm evap} g_{\rm cl} {\rm Mpc}^{-3}{\rm Myr}^{-1}$; they also universally imply these binaries are born equally often in the nearby and early universe [Fig. 2]. Finally, based on the weak scaling of merger rate with $\sigma_{\rm cl}$ implied by Peters' equations, the BH-BH merger rate depends weakly (empirically, roughly *logarithmically*) on the assumed initial cluster velocity dispersion $\sigma_{\rm cl}$ between 5km/s and 20km/s. In short, binary BHs merging due to this process do so at a relatively well-defined rate, agreeing to order of magnitude with the merger rate expected if all mass presently in stars formed 10 Gyr ago: $R_{\rm evap} \approx g_{\rm cl} g_{\rm evap} 2 \times 10^{-5} M_\odot^{-1}\rho_*/(10Gyr) \approx 0.8 g_{\rm evap} g_{\rm cl} {\rm Mpc}^{-3}{\rm Myr}^{-1}$.[54]

Unless suppressed strongly (i.e., $g_{\rm cl} g_{\rm evap} \ll 10^{-2}$), the BH-BH merger rate due to clusters will significantly exceed the average rate densities expected from isolated stellar evolution, $\approx 10^{-2}{\rm Mpc}^{-3}{\rm Myr}^{-1}$ [3, 39]. Additionally, as noted in O'Leary et al., the BH-BH binaries produced from dynamical cluster evaporation will strongly favor pairs of the highest-mass binaries (e.g., $14M_\odot + 14M_\odot$ or even $20M_\odot + 20M_\odot$). On the other hand, isolated binaries rely on SN kicks and mass transfer to bring them to merging; therefore these merging BH-BH binaries are typically significantly less massive. Because merging BH-BH binaries produced in clusters possess a distinctive mass signature and could occur at unusually high rates, GW observatories can directly constrain or even measure $g_{\rm cl} g_{\rm evap}$. For example, in an optimistic case ($g_{\rm cl} = g_{\rm evap} = 1$), the initial LIGO network could detect roughly 10 events per year, based on an estimated network range to $14M_\odot + 14M_\odot$ binaries of 125Mpc; the "enhanced LIGO" upgrade, with roughly twice the sensitivity, should see roughly $2^{15/6}$ as many sources; and with roughly $20\times$ the range, the advanced LIGO network could detect as many as $3 \times 10^4 g_{\rm cl} g_{\rm evap}$ sources per year, permitting exquisite probes of early-universe cluster dynamics if indeed this process yields more BH-BH mergers than isolated stellar evolution.[55]

Gravitational wave observatories provide useful information precisely because $g_{\rm cl}$ and $g_{\rm evap}$ are so weakly constrained electromagnetically. For example, based on galactic [5, 6] and extragalactic [7, 8, 12] cluster observations, the fraction of stars born in clusters $g_{\rm cl}$ could be anywhere from 20% to 100%. Similarly, observations of galactic and extragalactic clusters cannot rule out all clusters more massive than $3\times10^4 M_\odot$ and thus with more than 100 BHs undergoing runaway segregation and evaporation. While such clusters are exceedingly rare in number ($p(M)dM \propto M^{-2}$ [40, 41, 42]), they likely contain a significant fraction of all cluster-forming mass: assuming that clusters range in size from $30M_\odot$ to $10^7 M_\odot$, roughly 45% of all cluster-forming mass lies in these most massive clusters. While this fraction depends very weakly (logarithmically) on the limiting masses assumed for the cluster mass spectrum, a flatter mass function, as suggested by some observations [40, 43], would imply a significantly higher fraction of mass in these most massive clusters. Therefore, if all sufficiently massive clusters undergo segregation runaway, $g_{\rm evap}$ could be close to unity. On the other hand, these evaporated BH-BH binaries appear only if their clusters of origin survived long enough as bound objects for gravitational segregation to occur. Because gravitational segregation should occur much more rapidly than the age of observed long-lived clusters, such as the globular clusters of the Milky Way, $g_{\rm evap}$ must be larger than the corresponding fraction of a galaxy's mass: $g_{\rm evap} \gtrsim 10^{-4} - 10^{-3}$ [12]. On the other hand, gravitational segregation should at best compete with and more likely occur more slowly than "infant mortality," the tendency of roughly $70-90\%$ of young clusters to disrupt within their first $\sim 10\,$Myr due to photoionization- and SN -driven gas ejection [5, 12, 44, 45, 46, 47]. Based on only 10% of all clusters surviving a rapid "infant mortality" epoch and on 45% of all clusters being sufficiently massive for the Spitzer instability to occur, we expect $g_{\rm evap} \approx 5 \times 10^{-2}$. To summarize, we expect $g_{\rm cl} g_{\rm evap}$

could be as high as 1 (corresponding to 10 merger detections per year with initial LIGO); is likely 5×10^{-2} (1 event every two years with initial LIGO; 3 events per year with "enhanced" LIGO); and is very likely higher than 10^{-4} (producing merging BH-BH binaries slightly less frequently than isolated binary stars, but with systematically higher masses).

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[55] The factor relating the initial and advanced LIGO detection rates is not precisely geometrical (i.e., $20^{15/6}$) due to cosmological redshift of the emitted gravitational waves out of LIGO's sensitive band, as well as cosmological volume factors influencing the scale of the light cone near $z \approx 0.5$ [53]; see Eqs. (13-20) of O'Leary et al. for details [18].